Bundling, information and platform competition
Job Market Paper

Keke Sun*

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Abstract

This paper studies the impact of pure bundling and the level of consumer information about developer subscription prices on duopolistic platform competition. I find that, in the presence of asymmetric network externalities, pure bundling emerges as a profit-maximizing strategy when platforms subsidize the low-externality side (consumers) and make profits on the high-externality side (developers). Bundling can be used as a tool to enhance the "divide-and-conquer" nature of platform's pricing strategies, and is more effective in stimulating consumer demand the larger proportion of informed consumers. I also find that consumer information intensifies price competition. Consequently, bundling and more consumer information improve consumer welfare, but bundling is less likely to emerge as the fraction of informed consumers increases.

JEL classification: L11, L13, L42, D43, D84

Keywords: Two-sided platforms; Bundling; Passive expectations; Responsive expectations; Platform competition

*Universitat Autònoma de Barcelona and Barcelona Graduate School of Economics.
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1 Introduction

A smartphone operating system (OS) platform accommodates applications and makes the interactions between consumers and application developers possible. This paper is motivated by the phenomenal growth in the smartphone industry. As in other two-sided industries, an important decision for firms, with consequences for competition and welfare, is whether to bundle the operating system with the hardware. Apple is currently a major competitor both in the smartphone and the OS markets and its success in hardware has a significant bearing on its success in this industry (Kenney and Pon, 2011). Bundling with a best-selling handset certainly adds to the platform's appeal for consumers[1]. Such bundling practice is also common in industries like the video game industry, where major competitors like Sony and Microsoft bundle the OS platforms with their in-house consoles.

In this paper, we emphasize yet another characteristic of these industries: asymmetry of information of the different players. Developers are industry-insiders; they are usually informed about all subscription prices and have good predictions of participation decisions on both sides of the platform. In contrast, not every consumer knows the fixed fees or royalties that the platforms charge to developers. Similarly, newspaper readers may not be aware of how much the newspaper charges advertisers for listing ads.

The goal of this work is to develop a theoretical model to analyze the bundling strategy for platforms when information may be less than perfect for some users on different sides of the platform. We address two main research questions. First, when does a platform practice bundling as a profitable entry accommodation strategy? Second, how does the level of consumer information affect platform competition and the bundling decision?

We consider a two-stage game in which one of the platforms makes a strategic decision, namely, whether to bundle with an in-house handset, in addition to competing through adjustment of tactical variables which are subscription prices. In the first stage, this platform decides whether to bundle with its in-house handset[2], laying the groundwork for the competitive interactions down the line. In the second stage, the platforms decide subscription prices simultaneously, and competition takes place. We do not consider bundling to be an act of predation, but rather a commitment to an aggressive pricing strategy. The platform sells the bundle at a discount, relative to separate selling, to stimulate consumer and developer demand.

Within the framework of the Hotelling model, two platforms compete for single-homing consumers and multi-homing developers. Departing from the standard setting of full information and responsive expectations for all users in the two-sided markets literature, we

[1]The iPhone has been the top-selling mobile phone in the U.S. Source: `http://www.bizjournals.com/boston/blog/mass_roundup/2013/02/apple-top-selling-us-mobile-phone.html`, accessed September, 2014.

[2]We consider pure bundling. Under pure bundling, the handset and the access to platform A can only be purchased as a bundle.

assume that some consumers are uninformed about developer subscription prices and hold passive expectations about developer participation, whereas the remaining consumers and all developers are informed about all subscription prices and hold responsive expectations.

We find that, in the presence of asymmetric network externalities, price competition can lead to consumer prices being strategic substitutes when platform preferences are small relative to the benefits of attracting an additional consumer. Therefore, bundling, as a commitment to an aggressive pricing strategy, may be in the interest of the firm and detrimental to the rival when platforms subsidize the low-externality side (consumers) and make profits on the high-externality side (developers). Bundling can be used as a tool to enhance the "divide-and-conquer" nature of the pricing strategies. When consumers are heterogeneous with respect to the valuation of the handset, bundling can also emerge when consumer prices are strategic complements, even though there is no subsidy for participation. Bundling expands consumer demand for platform adoption as well as for the handset. Through bundling, the platform coordinates the misaligned consumer valuations of the platform and the handset, attracting consumers with a high valuation of the handset. Our results further show that bundling improves consumer welfare by lowering the prices and offering more application variety for the majority of consumers.

Our second set of findings concerns consumer information. We find that, when the fraction of informed consumers is larger, price competition is more intense. Informed consumers respond to price changes by adjusting their demand and their expectations of developer demands accordingly. Therefore, bundling, deployed as an implicit discount on the consumer subscription price, is more effective to stimulate consumer demand when there are more "responsive" consumers. Consequently, the developer demand is also stimulated through the demand shifting effect. Consumer surplus increases in the fraction of informed consumers, because a larger fraction of informed consumer leads to lower subscription prices and more application variety. Consumer information also has a negative impact on the emergence of bundling: the region in which bundling emerges shrinks as the fraction of informed consumers increases. This is because bundling only emerges when the competing platform increases its consumer price in response to bundling, but more consumer information intensifies competition and pushes the competing platform to be more aggressive.

The remainder of the paper is organized as follows. Section 2 briefly discusses the related literature. In Section 3, we set up the duopoly model of platform competition. In Section 4, we investigate the bundling strategy of the platform when consumers are homogeneous with respect to the valuation of the handset. We compare two scenarios, depending on whether the platform practices or does not practice bundling. Section 5 studies the bundling strategy when consumers are heterogeneous with respect to the valuation of the handset. Section 6 concludes.

2 Relationship to the Literature

This work contributes to the literature on two-sided platforms. A large share of this literature studies pricing in the presence of network effects[3]. The literature shows that the structure of equilibrium prices depends on the relative size of demand elasticities and indirect network externalities on each side, the marginal costs of serving each side, whether the market structure has single-homing users on each side or takes the form of a competitive bottleneck, that is having single-homing users on one side and multi-homing users on the other side. This works studies the impact of bundling in the framework of two-sided platforms. Pure bundling is usually considered as an act of predation. Whinston (1990) shows that pure bundling reduces equilibrium profits of all firms; hence, it is usually adopted to deter entry or drive the rival out of the market. However, in two-sided markets, this may not be the case. Our paper fits this theme by considering bundling as a tool to stimulate consumer demand. The present work is closely related to Farhi and Hagiu (2008) and Amelio and Jullien (2012). The former study shows that a subsidy on one side may lead to fundamentally new strategic configurations in oligopoly. Farhi and Hagiu (2008) present the conditions upon which a cost-reducing investment by intermediaries may be a successful entry accommodation strategy and may also benefit its rival. A possible interpretation is that this reduction results from a tying strategy. Prices are not necessarily strategic complements in competition, the effects of cost-reducing investments on prices are ambiguous and platforms may earn negative margin on one side. Amelio and Jullien (2012) investigate the effects of tying of independent goods, with single-homing users on both sides of the platform. With a non-negativity constraint, tying works as an implicit subsidy. In the monopoly case, the platform prefers tying as it is a way to subsidize users that have low network externality. Tying leads to higher participation, higher consumer surplus as well as profits. In the duopoly case, tying on one side makes a platform more or less competitive on the other side depending on externalities of the two sides. The impact of tying on platforms' profits also depends on the relative levels of externalities. Total consumer surplus increases in case of high asymmetry in the network externalities between two sides. Tying is used to implement second-degree price discrimination to help a network to coordinate the customers' participation. In Amelio and Jullien (2012), pure bundling arises if the value of the good is below its cost so that selling the good alone is not profitable. In this work, we show that pure bundling works as a commitment device to allow the platform to be more aggressive. The key features differentiating our work from this line of literature are: firstly, we study the effect of information asymmetry, namely, the level of consumer information, on platform's bundling strategy; secondly, we allow consumers to be heterogeneous with respect to the valuation of the handset. We believe that these features bring our analysis closer to reality. Using the one side single-homing and one side multi-homing, we show that bundling always hurt the rival, which differs from Farhi and Hagiu (2008) and Amelio and Jullien

[3]See Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), Armstrong and Wright (2007), Hagiu (2006), Weyl (2010).

(2012). We also show that without the non-negativity constraint, tying makes no difference from untying when consumers have the homogeneous valuation of the handset tied and the consumer market is fixed-sized.

Our work is also about information asymmetry across the two sides of the platform. The main characteristic of two-sided platforms is the bilateral indirect network externalities where one group's benefit from joining the platform depends on the size of the other group that joins the same platform, which gives rise to a "chicken-and-egg" problem (Armstrong, 2006; Hagiu, 2006). The majority of the existing literature on two-sided platform pricing assumes that all users have full information about all prices and preferences, which implies that all users can perfectly predict other's participation decision. In reality, platform users, especially consumers, may not be able to observe all prices or perfectly anticipate the impact of price changes on demands. Hurkens and López (2014) suggest that passive expectation should be a plausible alternative for responsive expectation in a market with (direct or indirect) network externalities. Passive expectations, first introduced by Katz and Shapiro (1985), are fulfilled in equilibrium. Consumers with passive expectations use price information on consumer's side to fixate their expectation of developer demand and do not respond to any price changes on the other side of the platform. In the present work, we allow users on the two sides to have different levels of information. We assume developers are always well-informed; they are informed about all prices and hold responsive expectations about consumer participation. Consumers are not necessarily as well-informed as developers, not every consumer is aware of how much the platforms charge developers for listing their applications. Therefore, we assume there is a fraction of consumers who are informed about developer prices and hold responsive expectations about developer participation while the remaining consumers are uninformed and hold passive expectations. In this spirit, the present paper is very close to Hagiu and Hałaburda (2014). They study the effect of different levels of information on two-sided platform profits, under both monopoly and competition. They assume that developers always hold responsive expectations while all users hold passive or responsive expectations. They show that responsive expectation amplifies the effect of price reductions. A monopoly platform can exploit the demand increases due to user's responsive expectation, so it prefers facing more informed users. While more information intensifies price competition, competing platforms are affected negatively when users are well informed. Our symmetric competition subgame is a replica of Hagiu and Hałaburda (2014)'s hybrid scenario in which some consumers are informed while others are uninformed and hold passive expectations. We reach the same conclusion that more information intensifies price competition regardless of the bundling decision. There is one key difference between Hagiu and Hałaburda (2014) and our analysis. They focus on the impact of different user expectations on equilibrium allocations in the context of monopoly and duopoly markets, whereas we are interested in the impact of the level of consumer information on platform's bundling decision in a duopoly setting, because our work models the competition between smartphone OS platforms where bundling has a significant bearing.

3 The Model

3.1 Platforms

Consider two platforms competing for both consumers and developers, indexed by T=A, B. Let p_T^C and p_T^D denote the subscription prices platform T charges to consumers and developers, respectively. We assume that the platforms have zero marginal cost of serving these two groups of users, which is consistent with the literature of information goods and the reality of digital media industry, where large fixed costs and very low marginal costs are observed[4]. We allow for negative prices, as it is possible for platforms to subsidize one side of the market. The number of consumers and developers on platform T are denoted by n_T^C and n_T^D, respectively. We allow single-homing on one side and multi-homing on the other side. To be more specific, we assume that each consumer decides in favor of only one platform while developers can design applications for both platforms.

We extend the standard Hotelling model by allowing the duopoly to serve two groups of users on each side of the market. The unit transportation cost for consumers towards each end is t, which is the platform differentiation parameter. Platform A and B are exogenously located at $x = 0$ and $x = 1$, respectively. Platform T's profit maximization problem is

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D,$$

where $T = A, B$.

We assume one of the platforms (without loss of generality, platform A) has a in-house killer handset with quality z and marginal cost C, for instance, the iPhone by Apple. For calculation simplicity, we normalize this marginal cost $C = 0$. Platform A is a monopolist in the high-end handset market, which vertically differentiated from the other handsets. Platform A can decide whether to bundle with this handset or not.

3.2 Consumers

There is mass 1 of consumers uniformly distributed along the unit interval, each of whom chooses at most one platform to join. The consumers have identical intrinsic values of two platforms, equivalent to v, which is assumed to be large enough so that the whole market is covered. Consumers have a taste for application variety. Every consumer's utility of participating on a platform depends on the total number of developers on the same platform. Consumers have identical utility gain from application variety; parameter θ is used to capture

[4]This assumption is made for calculation simplicity. Assuming platforms have the marginal cost of c^C and c^D of serving consumer's side and developer's side complicates the calculations but do not change the qualitative results of the model.

this direction of network externalities. More specifically, the availability of each additional developer positively generates additional utility θ for consumers, i.e. $\theta>0$. We ignore the potential positive direct externalities among consumers[5]. The consumer who locates at x decides joining platform A or B by comparing utilities $v+\theta n_A^{D^e}-p_A^C-tx$ from platform A and $v+\theta n_B^{D^e}-p_B^C-t(1-x)$ from platform B. Following Hagiu and Hałaburda (2014), we assume there are two types of consumers: a fraction λ of consumers is informed about developer subscription prices and holds responsive expectations about developer participation when choosing between two platforms, $0\leq\lambda\leq1$. The expectations of these consumers match the realized developer demand, i.e., $n_T^{D^e} = n_T^D$. The remaining fraction $1-\lambda$ of consumers is uninformed about developer prices and holds passive expectations. They do not adjust their expectation of developer demand in response to price changes on developer's side[6]. Therefore, the realized consumer demand of each platform is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t} + \frac{\lambda\theta n_T^D - \lambda\theta n_{-T}^D}{2t} + \frac{(1-\lambda)\theta n_T^{D^e} - (1-\lambda)\theta n_{-T}^{D^e}}{2t}, \tag{1}$$

where $T = A, B$.

For now, we assume consumers are homogeneous with respect to the valuation of the handset; they have identical marginal utility of the quality of platform A's in-house handset $\phi=1$, and each buys at most one copy.

3.3 Developers

There is mass 1 of potential developers; each developer lists one application on one platform. Assume that developers form responsive expectations of consumer demand. Developers are industry insiders, they are aware of consumer's preference, thus, can perfectly predict consumer participation. Developers differ in the cost of listing applications, denoted by y, and are uniformly distributed along the segment $[0,1]$. Each developer gains additional utility of β from each consumer who has access to its application. The revenue for a developer who lists on platform T is given by βn_T^C when the number of consumers who participate in platform T is n_T^C. We assume all applications are independent of one another, so the potential negative direct network externalities are ignored. The utility of developer y from joining platform T is

$$u_T^D = \beta n_T^C - p_T^D - y,$$

where $T = A, B$. We assume that developers can multi-home[7] and there are no economies of scope in multihoming. Therefore, the decision of joining a platform is independent of the

[5]The potential positive externalities indicate that consumers may derive positive utilities from the number of other consumers on the same platform.

[6]Hurkens and López (2014) offers a clear illustration of the difference between passive and responsive expectations.

[7]Some recent survey shows that on average mobile developers use 2.6 mobile platforms (VisionMobile, 2013).

joining decision of the other platform. That is, a y-type developer will join platform T if $u_T^D(y) = \beta n_T^C - p_T^D - y \geq 0$. So, the developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D.$$

Developers care more about the network benefits of reaching out to the widest population of consumers than they do about the cost of multi-homing since there is no standalone value for developers to join the platforms. We study a case of "competitive bottlenecks" (Armstrong, 2006): there is a high level of competition on consumer's side, and platforms make low profits on this side, but there is no competition for providing applications to consumers.

We assume that the following conditions hold throughout this paper:
Assumption A1. $\beta > 2\theta$.

We assume developers care more about consumers than consumers do about developers. This level of asymmetry between the two directions of network effect guarantees the existence of the situation where platforms engage in divide-and-conquer strategies.
Assumption A2. $t > \underline{t} = \frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2}$.

With these two assumptions, the conditions for unique and stable equilibrium ($t > \frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}$) and second order condition ($t > \theta\beta\lambda$) are satisfied, and both platforms make positive profits in equilibrium, so that they remain active in the market[8].

As we are interested in the impact of bundling on platform competition, we assume platform A's bundling decision cannot drive its rival out of the market:
Assumption A3. $0 < z < \overline{z} = 3t - 3\underline{t}$.

When $z \geq \overline{z}$, platform A would always bundle the platform with the handset to push the rival out of the market.

The next condition rules out the corner solution that the developer demand for each platform is 1:
Assumption A4. $\theta + \beta < 2$.

We propose a two-stage game. The timing of the game is as follows: In Stage 1, platform A makes the strategic decision whether to bundle with its in-house handset or not. The decision is publicly observable. In Stage 2, two platforms simultaneously decide on subscription prices for consumers and developers, and competition takes place.

[8]Notice that $\frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} - (\frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}) = (\frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6})(1-\lambda) \geq 0$ and $\frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} - \theta\beta\lambda = \frac{\beta^2}{6} + \frac{\theta\beta}{6} + \frac{\theta^2\lambda}{6} - \frac{\theta\beta\lambda}{2} \geq \frac{\beta^2\lambda}{6} + \frac{\theta\beta\lambda}{6} + \frac{\theta^2\lambda}{6} + \frac{\theta\beta\lambda}{2} = \frac{\lambda}{6}(\beta-\theta)^2 \geq 0$. Therefore, once Assumption $A2$ is satisfied, both $t > \frac{\beta^2}{6} + \frac{\theta^2\lambda^2}{6} + \frac{2\theta\beta\lambda}{3}$ and $t > \theta\beta\lambda$ hold.

4 Platform Competition

We analyze the bundling strategy of the platform by comparing the no bundling and bundling scenarios. We also compare bundling with tying. The difference between pure bundling and tying is that, the tied good is still available on a stand-alone basis under tying, which means that, under tying, consumers on platform B can still purchase the handset (see Tirole, 2005).

4.1 Symmetric Competition

We first derive the competition outcomes when platform A doesn't bundle with its in-house handset as the benchmark case. Platforms engage in symmetric competition as they both make profits through subscription. Platform T's profit function is given by

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D,$$

where $T = A, B$. Platform A also has revenue z stemming from the in-house handset.

A fraction λ of consumers are informed about all subscription prices; they make the participation decision upon subscription prices for both consumers and developers. The remaining consumers are only informed about prices on consumer's side; they make the participation decision upon consumer subscription prices and their expectations about developer participation for each platform. Thus, the realized consumer demand for each platform is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda}, \tag{2}$$

where $T = A, B$.

Proposition 1. *When two platforms engage in symmetric competition, the competition equilibrium outcomes are as follows:*

$$p_T^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4}, \qquad n_T^{C^*} = \frac{1}{2},$$

$$p_T^{D^*} = \frac{\beta}{4} - \frac{\theta\lambda}{4}, \qquad n_T^{D^*} = \frac{\beta}{4} + \frac{\theta\lambda}{4},$$

$$\pi_A^* = \frac{t}{2} - \frac{\theta^2\lambda^2}{16} - \frac{3\theta\beta\lambda}{8} - \frac{\beta^2}{16} + z,$$

and

$$\pi_B^* = \frac{t}{2} - \frac{\theta^2\lambda^2}{16} - \frac{3\theta\beta\lambda}{8} - \frac{\beta^2}{16},$$

where $T = A, B$.

Proof. See Appendix. □

The equilibrium consumer subscription price is the standard Hotelling price with zero marginal cost (t) adjusted downwards by $\frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$. The adjustment term, which measures the benefits to the platform from attracting an additional consumer, can be decomposed into two factors $\beta(\frac{\beta}{4} + \frac{3\theta\lambda}{4})$. The first factor β means the platform attracts β additional developers when it has an additional consumer. The second factor $\frac{\beta}{4} + \frac{3\theta\lambda}{4}$ is the profit that the platform can earn from an additional developer. The additional developer pays a subscription price $\frac{\beta}{4} - \frac{\theta\lambda}{4}$ to the platform, also attracts $\theta\lambda$ informed consumers because only informed consumers would adjust their expectations of developer demand according to price changes. The platforms decide their pricing strategies on consumer's side by comparing platform preferences with the benefits of attracting one extra consumer. The larger network externalities (β and θ) are, the lower price is charged on the consumer's side.

The equilibrium developer subscription price is the monopoly pricing $\frac{\beta}{4}$[9] adjusted downwards by $\frac{\theta\lambda}{4}$, where $\frac{\theta\lambda}{4}$ is the extra benefit that an extra developer brings to the platform from attracting informed consumers. When β is large, developers attach a high value to consumer participation, and platforms have incentives to lower consumer prices or even subsidize consumers for participation. So that the platforms can charge higher prices on developer's side. The equilibrium developer price increases with developer's network externalities. When θ is large, consumers attach a high value to developer participation, and platforms have incentives to lower developer prices to encourage participation, the equilibrium developer price decreases with consumer's network externalities.

Corollary 1. *The subscription prices on both sides of the platform are negatively affected by the fraction of informed consumers while developer participation is positively affected by it. The platform profits decrease in the fraction of informed consumers.*

When informed consumers are offered a lower price, they anticipate that consumer demand would increase and developer demand would increase accordingly. This intensifies price competition (Hagiu and Hałaburda, 2014). Indeed, the intensity of competition increases in the fraction of informed consumers.

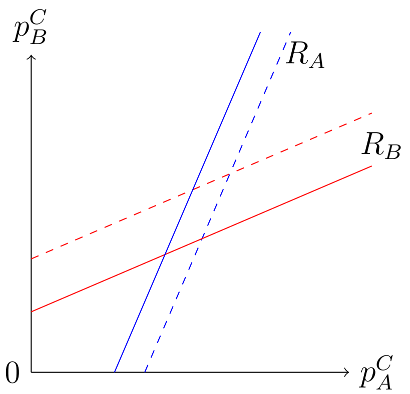
The best response function on consumer's side is

$$\begin{aligned} p_T^C(p_{-T}^C) = & \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t - \theta^2\lambda^2 - 3\theta\beta\lambda)}{\gamma} p_{-T}^C \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t^2 + \theta^2\beta^2\lambda^2 - 5t\theta\beta\lambda)}{\gamma} \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)\theta(1-\lambda)(4t - 3\theta\beta\lambda)(n_T^{D^e} - n_{-T}^{D^e})}{\gamma}, \end{aligned} \tag{3}$$

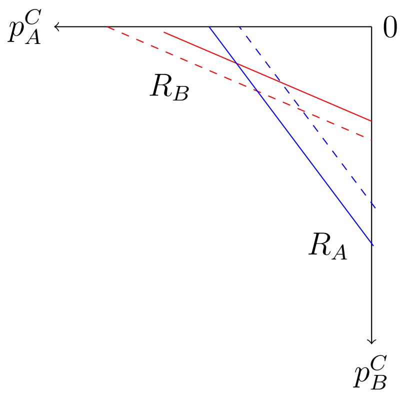
[9] If all consumers are uninformed about developer subscription prices and hold passive expectations about developer demand, platforms exploit monopoly power on developer's side and charge developer subscription price $\frac{\beta}{2} p_T^C$, the equilibrium developer subscription price is $\frac{\beta}{4}$.

where $\gamma = 32t^2 - 4t\theta^2\lambda^2 - 44t\theta\beta\lambda - 4t\beta^2 + 3\theta^3\beta\lambda^3 + 14\theta^2\beta^2\lambda^2 + 3\theta\beta^3\lambda$.

Depending on whether the platforms charge consumers positive prices or subsidize consumers for participation, we have two cases. When $t > \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the platforms charge consumers positive prices for participation, this is the case where platform preferences are larger than the benefits of attracting an extra consumer. The best response curves on consumer's side are upward-sloping (see the dashed lines in Figure 1(a)), the consumer prices of the two platforms are strategic complements ($\frac{\partial p_T^C(p_{-T}^C)}{\partial p_{-T}^C} > 0$). This is the case we often see in one-sided market. When $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$,platform preferences are small relative to the benefits of attracting an extra consumer. This is the case where platforms subsidize consumers, which is the low-externality side, and earns a positive margin on developer's side, which is the high-externality side. This is often seen in two-sided markets as the "divide-and-conquer" strategy(Caillaud and Jullien, 2003). The best response curves on consumer's side are downward-sloping (see the dashed lines in Figure 1(b)), and the consumer prices are strategic substitutes ($\frac{\partial p_T^C(p_{-T}^C)}{\partial p_{-T}^C} < 0$) (Besanko et al., 2000).



(a) Strategic Complements



(b) Strategic Substitutes

Figure 1: Best response curves on consumer side under bundling

4.2 Bundling

We assume platform A sets p_A as the price for the bundled products, then $p_A^C = p_A - z$ is the implicit subscription price for consumers. Under pure bundling, neither the in-house handset nor the access to platform A would be available on a standalone basis. Platform A would now charge a lower price for the bundled products, relative to separate selling. Under bundling, platform A has more incentives to lower the consumer price, a fall of p_A^C not only encourages consumer participation, but also stimulates demand of the handset. The marginal consumer locating at x derives utility $v + z - (p_A^C + z) - tx + \theta n_A^{D^e}$ from purchasing the bundle and $v - p_B^C - t(1-x) + \theta n_B^{D^e}$ from joining platform B. Again, a fraction λ of consumers is informed about developer subscription prices and holds responsive expectations about developer participation, i.e., $n_T^{D^e} = n_T^D$; while the remaining consumers are uninformed and hold passive expectations. Therefore, the consumer demand of each platform is the same as Eq. (1). Platform A's profit maximization problem evolves to

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D = (p_A^C + z) n_A^C + p_A^D n_A^D.$$

Platform B's profit maximization problem is unchanged.

The following proposition characterizes the equilibrium prices and allocations in the bundling scenario.

Proposition 2. *When platform A bundles with its in-house handset and consumers are homogeneous with respect to the valuation of the handset, the equilibrium outcomes are as follows:*

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(8t - 2\theta\beta - \beta^2 - 2\theta^2\lambda - 3\theta\beta\lambda)}{12(t - \underline{t})},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{12(t - \underline{t})},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{6(t - \underline{t})}, \qquad n_B^{C^*} = \frac{1}{2} - \frac{z}{6(t - \underline{t})},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6(t - \underline{t})}), \qquad n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6(t - \underline{t})}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6(t - \underline{t})}), \qquad n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6(t - \underline{t})}),$$

$$\pi_A^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16} \frac{(6(t - \underline{t}) + 2z)^2}{36(t - \underline{t})^2}$$

and

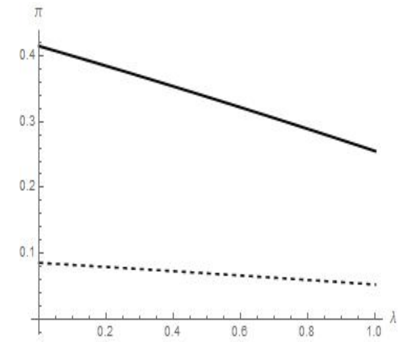
$$\pi_B^* = \frac{8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda}{16} \frac{(6(t - \underline{t}) - 2z)^2}{36(t - \underline{t})^2}.$$

Proof. See Appendix. □

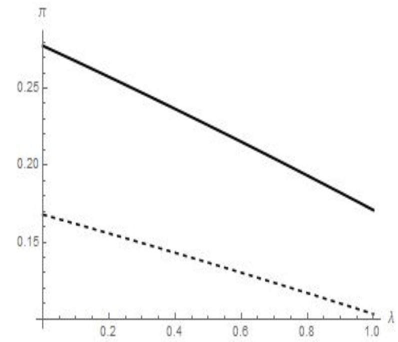
Under bundling, only consumers on platform A purchase the handset. Platform A has to lower its consumer price to stimulate the demand for the bundled products. Thus, it manages to steal some consumers from its rival. The higher value of the bundling handset, the more leverage platform A has on consumers. However, the cut on platform A's consumer price dominates the increment on its consumer demand. Hence, platform A suffers a loss on consumer's side. Yet, the direction of change on platform B's consumer subscription price is ambiguous, depending on the strategic relationship between consumer subscription prices. When consumer prices are strategic complements (resp. substitutes), platform B's consumer price goes down (resp. up). The directions of changes on profits from developer's side for both platforms are clear. As a fall on p_A^C shifts the consumer demand toward platform A, platform A (resp. B) becomes more (resp. less) attractive on developer's side through network effects. This effect increases with β, which determines the sensitivity of developer demand to the demand on consumer's side.

Corollary 2. *When platform A bundles with its in-house handset, its implicit consumer price decreases with the fraction of informed consumers while its demands on both sides of the platform increase with it. Platform B's developer price, demands on both sides of the platform and total profits are negatively affected by the fraction of informed consumers.*

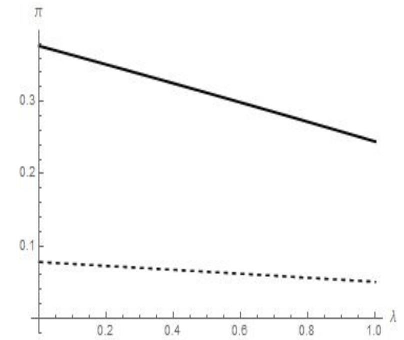
This corollary has significant empirical implications. It indicates that bundling is a more effective tool to stimulate consumer demand when there are more informed consumers. The effect of a discount on platform A's consumer price is amplified. Platform A's demands on both sides of the market reach the highest levels when all consumers are informed. Platform B suffers the largest loss when all consumers are informed. The effect of the fraction of the informed consumers on platform A's profits is ambiguous. Figure 2 illustrates how platform profits under bundling change with the fraction of informed consumers λ, for certain values θ, β and z. All graphs have parameter $t = 0.5$. The solid line depicts π_A and the dotted line depicts π_B.



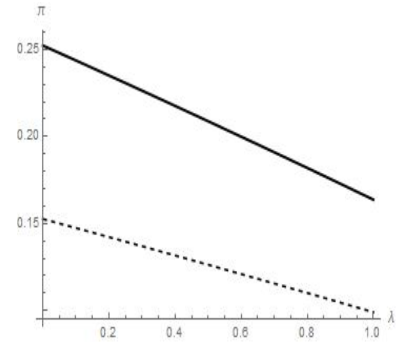
(a) $\theta = 0.3$, $\beta = 0.7$, $z = \frac{3}{4}(3t - 3\underline{t})$



(c) $\theta = 0.3$, $\beta = 0.7$, $z = \frac{1}{4}(3t - 3\underline{t})$



(b) $\theta = 0.2$, $\beta = 0.9$, $z = \frac{3}{4}(3t - 3\underline{t})$



(d) $\theta = 0.2$, $\beta = 0.9$, $z = \frac{1}{4}(3t - 3\underline{t})$

Figure 2: Platform profits as functions of the level of consumer information

The system of best response functions on consumer's side is as follows:

$$\begin{aligned} p_A^C(p_B^C) = & \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t - \theta^2\lambda^2 - 3\theta\beta\lambda)}{\gamma} p_B^C \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)(4t^2 + \theta^2\beta^2\lambda^2 - 5t\theta\beta\lambda)}{\gamma} \\ & + \frac{(4t - \beta^2 - 3\theta\beta\lambda)\theta(1 - \lambda)(4t - 3\theta\beta\lambda)(n_A^{D^e} - n_B^{D^e})}{\gamma} \\ & - \frac{16t^2 - 4t\theta^2\lambda^2 - 20t\theta\beta\lambda + 3\theta^3\beta\lambda^3 + 5\theta^2\beta^2\lambda^2}{\gamma} z, \end{aligned} \tag{4}$$

and

$$
\begin{aligned}
p_B^C(p_A^C) = & \frac{(4t-\beta^2-3\theta\beta\lambda)(4t-\theta^2\lambda^2-3\theta\beta\lambda)}{\gamma} p_A^C \\
& + \frac{(4t-\beta^2-3\theta\beta\lambda)(4t^2+\theta^2\beta^2\lambda^2-5t\theta\beta\lambda)}{\gamma} \\
& + \frac{(4t-\beta^2-3\theta\beta\lambda)\theta(1-\lambda)(4t-3\theta\beta\lambda)(n_B^{D^e}-n_A^{D^e})}{\gamma} \\
& - \frac{\theta^2\lambda^2(4t-\beta^2-3\theta\beta\lambda)}{\gamma} z.
\end{aligned} \tag{5}
$$

Compared to Eq. (3), there are two effects determining the movements of the best response curves. The terms proportional to z represent the impact of bundling on consumer prices. Bundling has a direct impact on consumer prices: all consumers observe the changes on consumer prices. It also has an indirect impact on consumer prices: informed consumers anticipate the impact of bundling on developer's participation decisions. The terms proportional to $n_A^{D^e} - n_B^{D^e}$ represent the impact of bundling on perceived platform quality in terms of application variety for uninformed consumers. Following Amelio and Jullien (2012), we separate the impact of p_A^C on the derivative of platform profits as follows:

$$
\frac{\partial}{\partial p_A^C}\left(\frac{\partial \pi_A}{\partial p_A^D}\right) = \frac{\partial n_A^C}{\partial p_A^D} + \frac{\partial n_A^D}{\partial p_A^C} = -\frac{\theta\lambda}{2t-2\theta\beta\lambda} - \frac{\beta}{2t-2\theta\beta\lambda}, \tag{6}
$$

and

$$
\frac{\partial}{\partial p_A^C}\left(\frac{\partial \pi_B}{\partial p_B^D}\right) = \frac{\partial n_B^D}{\partial p_A^C} = \frac{\beta}{2t-2\theta\beta\lambda}. \tag{7}
$$

The term $-\frac{\beta}{2t-2\theta\beta\lambda}$ in Eq. (6) captures the fact that a fall of p_A^C shifts the consumer demand towards platform A. As a result, platform A becomes more attractive for developers. The best response curve of platform A shifts upwards because its perceived quality has improved for consumers. Similarly, the term $\frac{\beta}{2t-2\theta\beta\lambda}$ in Eq. (7) indicates that a fall of p_A^C makes platform B less attractive for developers.

The term $-\frac{\theta\lambda}{2t-2\theta\beta\lambda}$ in Eq. (6) captures the other direction of the demand shifting effect. A fall of p_A^D increases the developer demand for platform A, which improves platform quality in terms of application variety. Therefore, the consumer demand shifts upwards. Note that this direction of effect is discounted because only informed consumers adjust their expectations of developer demand according to the price change. The sensitivity of this direction of demand shifting effect depends on both consumer's network externalities and the fraction of informed consumers. The higher fraction of informed consumers there is, the more sensitive this direction of demand shifting effect is.

In the same fashion as before, we further discuss the impact of bundling decision depending on whether the platforms charge consumers positive prices or subsidize consumers for participation.

4.2.1 Case I. Strategic Complements

When $t > \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the best response curves are again upward-sloping, indicating that the consumer prices are strategic complements. The best response curves are shown in Figure 1(a). Compare to the dashed lines in Figure 1(a), we see that, under bundling, the response curve of platform A moves to the left and the curve of platform B shifts downwards. Through bundling, platform A offers a discount on consumer subscription price, so the response curve of platform A moves downwards, but this effect is dampened by consumer's expectations of more application variety on platform A. Under bundling, platform A has a higher consumer demand, the demand shifting effect indicates that it also has a higher developer demand, the perceived quality of platform A increases and the perceived quality of platform B decreases. Platform A's best response curve has the tendency to move upwards. Also, when consumer prices are strategic complements, platform B lowers its price in response to bundling.

Pure bundling, works as a commitment device, has both a direct and a strategic effect on the platform's profits (Besanko et al., 2000). The direct effect of the commitment is its impact on the platform profits if the rival's behavior does not change, and the strategic effect takes into account how the commitment changes the tactical decisions of rivals and, ultimately, the market equilibrium (Besanko et al., 2000). We decompose the effect of z on platform A's own profits into a direct effect and strategic effects on both sides of the platform.

$$\frac{d\pi_A}{dz} = \frac{\partial \pi_A}{\partial z} + \frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C*}}{dz} + \frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D*}}{dz}$$

Note that the direct effect is $\frac{\partial \pi_A}{\partial z} = n_A^C$. It indicates that platform A suffers a loss on the handset sales under bundling compared to the no bundling case. The term $\frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C*}}{dz}$ represents the strategic effect of bundling on consumer's side:

$$\frac{\partial \pi_A}{\partial p_B^C}\frac{dp_B^{C*}}{dz} = (p_A^C + z + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^C}\left(-\frac{4t - \beta^2 - 3\theta\beta\lambda}{12(t - \underline{t})}\right) < 0.$$

The intuition is that bundling drives the rival to set the consumer price low when prices are strategic complements, it intensifies competition on consumer's side. The last term $\frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D*}}{dz}$ represents the strategic effect of bundling on developer's side:

$$\frac{\partial \pi_A}{\partial p_B^D}\frac{dp_B^{D*}}{dz} = (p_A^C + z + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^D}\left(-\frac{\beta - \theta\lambda}{12(t - \underline{t})}\right) < 0.$$

Bundling has a strategic effect on developer's side because informed consumers adjust their expectations about developer participation due to bundling. Bundling makes the competing platform less attractive for developers through the demand shifting effect. Consequently, the competing platform has to lower its developer price. In this case, bundling intensifies competition on both sides of the platform. The speed of platform A's profit increasing in the value of the handset drops from 1 (no bundling) to a speed slower than n_A^C (under bundling) (see Figure 3(a)). Bundling cannot be profitable for platform A in this case.

Bundling has strategic effects on platform B's profits:

$$\frac{d\pi_B}{dz} = \underbrace{\frac{\partial \pi_B}{\partial z}}_{=0} + \frac{\partial \pi_B}{\partial p_A^C}\frac{dp_A^{C*}}{dz} + \frac{\partial \pi_B}{\partial p_A^D}\frac{dp_A^{D*}}{dz}.$$

The first term of strategic effect also concerns the effect of bundling on consumer's side:

$$\frac{\partial \pi_B}{\partial p_A^C}\frac{dp_A^{C*}}{dz} = (p_B^C \frac{\partial n_B^C}{\partial p_A^C} + p_B^D \beta \frac{\partial n_B^C}{\partial p_A^C})(-\frac{8t - \beta^2 - 2\theta\beta - 2\theta^2\lambda - 3\theta\beta\lambda}{12(t - \underline{t})}) < 0.$$

Under bundling, platform A sets a low subscription price, platform B has to lower its price in response. Platform A's bundling decision leads to a more competitive environment on consumer's side. On developer's side, the strategic effect of bundling is:

$$\frac{\partial \pi_B}{\partial p_A^D}\frac{dp_A^{D*}}{dz} = (p_B^C \frac{\partial n_B^C}{\partial p_A^D} + p_B^D \beta \frac{\partial n_B^C}{\partial p_A^D})(\frac{\beta - \theta\lambda}{12(t - \underline{t})}) > 0.$$

Bundling makes platform A more attractive to developers, which increase its developer subscription price. Thus, there is room for platform B to increase its developer subscription price as well. Bundling softens competition on this side of the platform. The over all effect of z on platform B's profits are as follows:

$$\frac{d\pi_B}{dz} = n_B^C(-\frac{8t - \beta^2 - 2\theta\beta - 2\theta^2\lambda - 3\theta\beta\lambda}{12(t - \underline{t})}) + \theta\lambda n_B^C(\frac{\beta - \theta\lambda}{12(t - \underline{t})}) < 0.$$

Bundling is detrimental to platform B's profit in this case.

4.2.2 Case II. Divide-and-Conquer

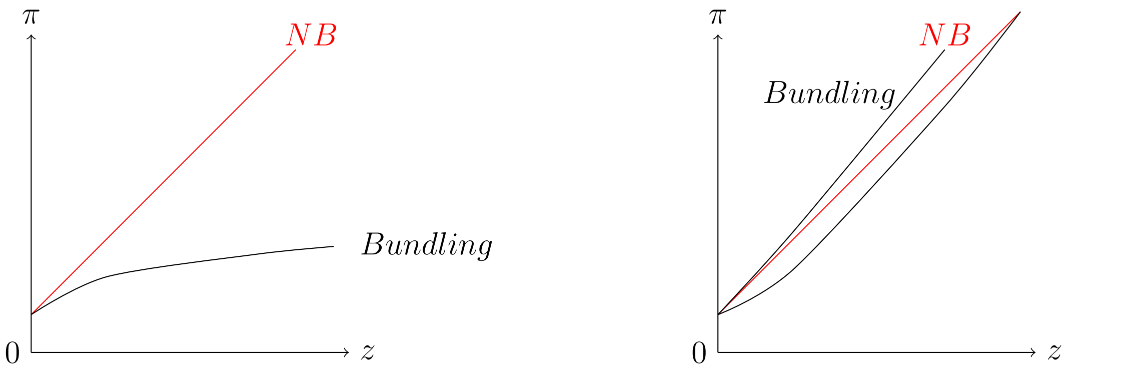
When $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\theta\beta\lambda}{4}$, the best response curves are downward-sloping and consumer subscription prices are again strategic substitutes. Both platforms subsidize consumers for participation. The changes on equilibrium consumer prices are shown in Figure 1(b). Compare to the dashed lines in Figure 1(b), the response curve of platform A shifts downwards under bundling. Through bundling, platform A increases the subsidy for consumer participation, so the response curve of platform A moves downwards. In response, platform B reduces its consumer subsidy because consumer prices are strategic substitutes. Platform B's best response curve moves upwards. The demand shifting effect indicates that platform A has higher developer participation under bundling. The perceived quality of platform A increases, platform A increases subsidy for consumer participation to compete very fiercely for consumer demand because the benefit of attracting one consumer is larger than platform preferences. The best response curve of platform A moves downwards further. Uninformed consumers expect platform B to offer less application variety, the perceived quality of platform B drops. The demand shifting effect indicates that platform B is less attractive to developers. Platform B cuts subsidy for consumer participation further, its best response curve moves upwards further.

We investigate the impact of z on platform A's profits:

$$\frac{d\pi_A}{dz} = n_A^C + n_A^C \frac{-4t + \beta^2 + 2\theta\beta\lambda + \theta^2\lambda^2}{12(t - \underline{t})}.$$

The strategic effect on consumer's side is positive. platform B's response to bundling is to reduce its consumer subsidy, which softens competition on this side of the platform. When platform preferences are small, it is possible that the positive strategic effect on consumer's side dominates the negative strategic effect on developer's side. Although the handset is only sold to consumers on platform A under bundling, platform A's profit increases in the value of the handset faster than n_A^C when consumer prices are strategic substitutes. As depicted in Figure 3(b), the speed of platform A's profits increasing in the value of the handset could be strictly faster than 1, which means that bundling would be profitable for any value of z, or bundling could be profitable only when the value of the handset is significant. In all, bundling can be profitable when consumers prices are strategic substitutes.

The overall effect of z on platform B's profits is negative, platform B cannot gain any profits from the rival's bundling practice.



(a) Strategic Complements

(b) Strategic Substitutes

Figure 3: The impact of bundling on platform profits

Platform A determines its bundling strategy by comparing the profits in two subgames. Let $t_1 = \frac{3\beta^2+4\theta\beta+6\theta\beta\lambda+4\theta^2\lambda-\theta^2\lambda^2}{16}$, $z_1 = \frac{(6t-6\underline{t})(16t+\theta^2\lambda^2-4\theta^2\lambda-6\theta\beta\lambda-4\theta\beta-3\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$ and $t_2 = \frac{5\beta^2+8\theta\beta+6\theta\beta\lambda+8\theta^2\lambda-3\theta^2\lambda^2}{24}$. The following proposition states platform A's bundling strategy.

Proposition 3. *When consumers are homogeneous with respect to the valuation of platform A's in-house handset,*
(i) platform A always chooses to bundle with the handset for all $z < \overline{z}$ *when* $\underline{t} < t \leq t_1$*;*
(ii) platform A bundles if the value of the handset is high, i.e., $z_1 \leq z < \overline{z}$*, when* $t_1 < t \leq t_2$*;*
(iii) platform A never practices bundling when $t > t_2$*,*
Bundling always hurts the rival.

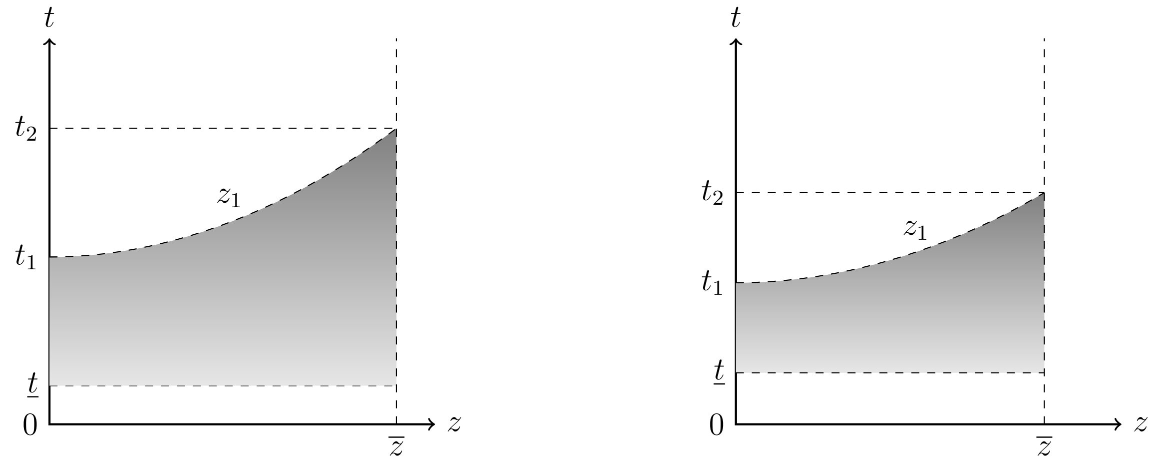
Proof. See Appendix. □

It is worth commenting that when platform preferences are small relative to the network externalities, a small extra consumer demand can lead to significant profits on developer's side, so platform A is willing to bundle with the handset even if it can only steal small consumer demand from the rival. When the platform preferences are medium, to recoup the loss on consumer's side due to bundling, platform A needs to have a great consumer demand. Therefore, platform A would practice bundling only when bundling can steal a large consumer demand from the rival, that is to say, the value of bundled handset needs to be significant. When platform preferences are large relative to the network externalities, platform A can never recoup the loss on consumer's side given a fixed-sized consumer market; bundling never occurs. Bundling works as a commitment to an aggressive pricing strategy and it only emergence when platforms subsidize consumers for participation, therefore, bundling can be used as tool to enhance the "divide-and-conquer" nature of the pricing strategies.

The following corollary reveals the impact of the level of consumer information on the bundling strategy.

Corollary 3. *The set of parameters upon which bundling emerges shrinks as the fraction of informed consumers increases.*

Bundling emerges only when the platforms engage in divide-and-conquer strategies, where the competing platform reduces its consumer subsidy in response to bundling. However, a larger fraction of informed consumers intensifies competition, pushing the competing platform to increase its consumer subsidy. In Figure 4, the grey area represents the region in which bundling would emerge. Bundling is less likely to occur when there is a large fraction of informed consumers.



(a) Bundling strategy: low λ

(b) Bundling strategy: high λ

Figure 4: The impact of consumer information on bundling strategy

4.3 Welfare Analysis

Now we address the issue how platform A's bundling practice affects consumer surplus. The equilibrium consumer surplus in two scenarios are as follows:

$$\begin{aligned} CS_{symmetric} &= \int_0^{n_A^{C^*}} (v + \theta n_A^{D^*} - tx - p_A^{C^*})dx + \int_{1-n_B^{C^*}}^{1} (v + \theta n_B^{D^*} - t(1-x) - p_B^{C^*})dx \\ &= v - \frac{5t}{4} + \frac{\beta^2}{4} + \frac{\theta\beta}{4} + \frac{\theta^2\lambda}{4} + \frac{3\theta\beta\lambda}{4}. \end{aligned}$$

$$\begin{aligned} CS_{bundling} &= \int_0^{n_A^{C^*}} (v + \theta n_A^{D^*} - tx + z - z - p_A^{C^*})dx + \int_{1-n_B^{C^*}}^{1} (v + \theta n_B^{D^*} - t(1-x) - p_B^{C^*})dx \\ &= v - \frac{5t}{4} + \frac{\beta^2}{4} + \frac{\theta\beta}{4} + \frac{\theta^2\lambda}{4} + \frac{3\theta\beta\lambda}{4} + \frac{z}{2} + \frac{tz^2}{(6t - 6\underline{t})^2} \end{aligned}$$

Corollary 4. *Consumer surplus is positively affected by the fraction of informed consumers. Under bundling, consumer surplus is positively affected by the value of the handset.*

A higher level of consumer information leads to lower subscription prices and higher developer participation, resulting in greater consumer surplus. Also, both consumer and developer participation on platform A is positively affected by the value of the bundling handset while platform A's consumer subscription price is negatively affected. The surplus of the majority of consumers increases with the value of the handset. Therefore, in general, consumer surplus increases with it.

Bundling has indeed one negative and two positive effects on consumer welfare. On the one hand, the unequal-split of consumer demand between two platforms increases total transportation cost, which reduces consumer welfare. The larger the difference in consumer demand between the two platforms, the larger adverse welfare effect of bundling. On the other hand, there are two positive welfare effects of bundling coming from the fact that the majority of consumers enjoy a lower subscription price and more application variety, which dominates the negative effect on consumer surplus caused by lower subsidy and less application variety for consumers on platform B. The change on consumer surplus due to platform A's bundling decision is

$$\begin{aligned}\Delta CS &= CS_{bundling} - CS_{symmetric} \\ &= \frac{z}{2} + \frac{tz^2}{(6t - 6\underline{t})^2} > 0\end{aligned}$$

Proposition 4. *When consumers are homogeneous with respect to the valuation of the bundling handset, platform A's bundling decision unambiguously improves consumer welfare.*

4.4 Tying

If platform A practices tying, it still sells the handset to consumers on platform B and extracts full surplus of the handset from them. Platform A's maximization problem now evolves to

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + z)n_A^C + p_A^D n_A^D + n_B^C z = p_A^C n_A^C + p_A^D n_A^D + z.$$

Proposition 5. *When consumers are homogeneous with respect to the valuation of platform A's in-house handset and platform A extracts full surplus from the fixed-sized handset market, tying makes no difference from untying.*

5 Extension: Heterogeneous Consumer Valuation of the Handset

Now we modify our setting regarding consumer's valuation of the handset. Let consumer's location on the unit interval be x and the marginal utility of the quality of platform A's in-house handset be ϕ. The pair (x, ϕ) defines a consumer type. Both x and ϕ are distributed independently and uniformly on $[0, 1]$. Type-ϕ consumer's utility from the handset is

$$U^{hs} = \phi z - p^{hs}.$$

Without bundling, platform A sells the handset at monopoly price $p^{hs} = \frac{z}{2}$, and the demand for this handset is $D(p^{hs}) = \frac{1}{2}$. Consumers with high marginal utility $\phi \geq \overline{\phi} = \frac{1}{2}$ purchase (Figure 1.5(a)). Platform A earns revenue $\pi^{hs} = \frac{z}{4}$ from the unbundled handset.

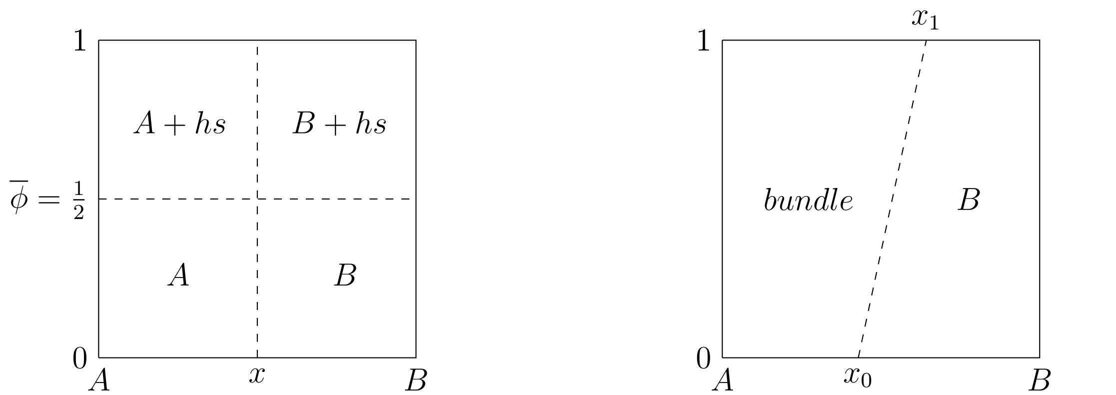
Again, we assume platform A sets p_A as the price for the bundled products, where $p_A = p_A^C + \frac{z}{2}$. Consumers with the heterogeneous marginal utility of the handset quality derive different levels of utility from purchasing the bundled products (Figure 1.5(b)). For instance, consumer $(x, 0)$ derives utility $v - p_A + \theta n_A^{D^e} - tx$ from purchasing the bundle and $v - p_B^C + \theta n_B^{D^e} - t(1-x)$ from joining platform B, the marginal consumer with 0 marginal utility for the handset quality is

$$x_0 = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} - \frac{z}{4t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

Similarly, consumer $(x, 1)$ derives utility $v - p_A + z + \theta n_A^{D^e} - tx$ from purchasing the bundle and $v - p_B^C + \theta n_B^{D^e} - t(1-x)$ from joining platform B, the marginal consumer with the highest marginal utility of the handset quality is

$$x_1 = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \frac{z}{4t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

The realized consumer demand of each platform is the same as Eq. (1.1).



(a) No Bundling

(b) Bundling

Figure 5: Consumers are heterogeneous with repsect to the valuation of the handset

Platform A's profit maximization problem evolves to

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D = (p_A^C + \frac{z}{2}) n_A^C + p_A^D n_A^D.$$

Proposition 6. *When platform A bundles with its in-house handset and consumer's marginal utility of the handset quality is uniformly distributed over* $[0,1]$, *the equilibrium outcomes are as follows:*

$$p_A^{C^*}=t-\frac{\beta^2}{4}-\frac{3\theta\beta\lambda}{4}-\frac{z(8t-2\theta\beta-\beta^2-2\theta^2\lambda-3\theta\beta\lambda)}{24(t-\underline{t})},$$

$$p_B^{C^*}=t-\frac{\beta^2}{4}-\frac{3\theta\beta\lambda}{4}-\frac{z(4t-\beta^2-3\theta\beta\lambda)}{24(t-\underline{t})},$$

$$n_A^{C^*}=\frac{1}{2}+\frac{z}{12(t-\underline{t})},\qquad n_B^{C^*}=\frac{1}{2}-\frac{z}{12(t-\underline{t})},$$

$$p_A^{D^*}=\frac{\beta-\theta\lambda}{2}(\frac{1}{2}+\frac{z}{12(t-\underline{t})}),\qquad n_A^{D^*}=\frac{\beta+\theta\lambda}{2}(\frac{1}{2}+\frac{z}{12(t-\underline{t})}),$$

$$p_B^{D^*}=\frac{\beta-\theta\lambda}{2}(\frac{1}{2}-\frac{z}{12(t-\underline{t})}),\qquad n_B^{D^*}=\frac{\beta+\theta\lambda}{2}(\frac{1}{2}-\frac{z}{12(t-\underline{t})}),$$

$$\pi_A^*=\frac{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}{16}\frac{(6t-6\underline{t}+z)^2}{(6t-6\underline{t})^2},$$

and

$$\pi_B^*=\frac{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}{16}\frac{(6t-6\underline{t}-z)^2}{(6t-6\underline{t})^2}.$$

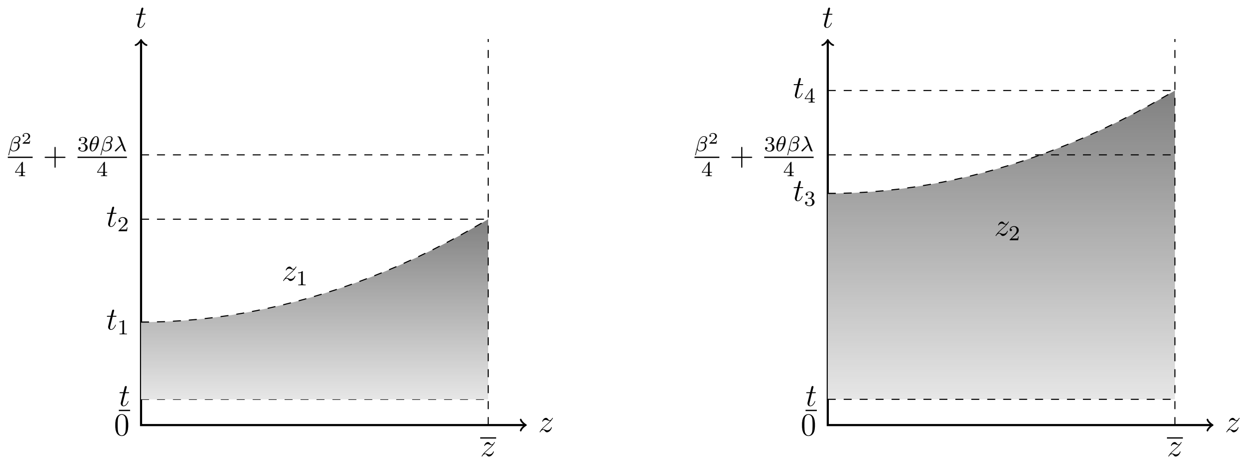
Proof. See Appendix. □

Let $t_3=\frac{2\theta\beta+\beta^2+2\theta^2\lambda-\theta^2\lambda^2}{4}$, $z_2=\frac{(6t-6\underline{t})(8t-4\theta\beta-2\beta^2-4\theta^2\lambda+2\theta^2\lambda^2)}{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}$ and $t_4=\frac{8\theta\beta+3\beta^2+8\theta^2\lambda-5\theta^2\lambda^2-6\theta\beta\lambda}{8}$. We use the following proposition to identify the bundling strategy for platform A when consumer's valuation of the handset is uniformly distributed along $[0,1]$.

Proposition 7. *When consumer's marginal utility of the quality of platform A's in-house handset is uniformly distributed over* $[0,1]$,
(i) platform A chooses to bundle with the handset for all $z<\overline{z}$ *when* $\underline{t}<t\leq t_3$*;*
(ii) platform A practices bundling iff $z_2\leq z<\overline{z}$ *when* $t_3<t\leq t_4$*;*
(iii) platform A never practices bundling when $t>t_4$*,*
Bundling always hurts the rival.

Proof. See Appendix. □

The set of parameters upon which bundling emerges is strictly larger than the case where consumers are homogeneous with respect to the valuation of the handset. Again, the set of parameters upon which bundling emerges shrinks as the fraction of informed consumers increases. Notice that bundling can be profitable even when consumer subscription prices are strategic complements. When all consumers are uninformed and hold passive expectations, bundling can be profitable even when consumer subscription prices are strategic complements regardless of the value of the handset. We compare the regions in which bundling emerges when consumers are homogeneous or heterogeneous with respect to the valuation of the handset (see Figure 1.6).



(a) homogeneous valuation of the handset

(b) heterogeneous valuation of the handset

Figure 6: Bundling strategy when consumers are homogeneous and heterogeneous with respect to the valuation handset

Indeed, the overall effect of z on platform A's profit is:

$$\frac{d\pi_A}{dz} = n_A^C + (p_A^C + \frac{z}{2} + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^C}(-\frac{4t - \beta^2 - 3\theta\beta\lambda}{12(t - \underline{t})}) + (p_A^C + \frac{z}{2} + \beta p_A^D)\frac{\partial n_A^C}{\partial p_B^D}(-\frac{\beta - \theta\lambda}{24(t - \underline{t})}).$$

When consumer prices are strategic substitutes, the strategic effects are positive, $\frac{d\pi_A}{dz} > n_A^C$, platform A's profit increases in value of the handset at a rate faster than n_A^C, also bundling expands consumer demand ($n_A^C > \frac{1}{2}$), bundling is profitable. When consumer prices are strategic complements, although platform A's profit increases in value of the handset at a rate slower than n_A^C, given the expanded consumer demand for the bundle, bundling still can be profitable.

From Figure 1.5(b), we see a difference in consumer demand between consumers with a high valuation of the handset and the ones with a low valuation. In fact, under bundling, more consumers with a high valuation of the handset ($\phi \geq \overline{\phi} = \frac{1}{2}$) join platform A than the ones with a low valuation ($\Delta n_A^C = n_{A(\phi \geq \overline{\phi})}^C - n_{A(\phi < \overline{\phi})}^C = \frac{z}{8t}$), and the differnce in demand increases with the value of the handset. Through bundling, platform A coordinates the misaligned consumer valuations of the platform and the handset, targeting consumers with a high valuation of the handset for participation.

5.1 Tying

If platform A decides to practice tying instead of bundling, the handset is still available to consumers on platform B. Among these consumers, only those with high marginal utility of the handset quality would purchase, i.e., $\phi \geq \overline{\phi} = \frac{1}{2}$. So, consumers with high marginal utility of the handset quality make participation decision by comparing the utility of buying the bundle from platform A with buying the access to platform B plus the handset from platform A (see Figure 8). The marginal consumer with high marginal utility of the handset quality locates at

$$x_{\phi \geq \overline{\phi}} = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}.$$

The consumers with low marginal utility of the handset quality have a demand

$$n^C_{A(\phi < \overline{\phi})} = \frac{1}{2} - \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

for the bundled products and

$$n^C_{B(\phi < \overline{\phi})} = \frac{1}{2} + \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \frac{\lambda\theta n_A^D - \lambda\theta n_B^D}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

for the access to platform B. Therefore, the realized consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} - \frac{z}{16t} + \frac{\lambda\theta n_A^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_A^{D^e} - (1-\lambda)\theta n_B^{D^e}}{2t}$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \frac{z}{16t} + \frac{\lambda\theta n_B^{D^e} - \lambda\theta n_B^{D^e}}{2t} + \frac{(1-\lambda)\theta n_B^{D^e} - (1-\lambda)\theta n_A^{D^e}}{2t}.$$

Platform A's profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = p_A n_A^C + p_A^D n_A^D + \frac{z}{2} n^{hs} = (p_A^C + \frac{z}{2}) n_A^C + p_A^D n_A^D + \frac{z}{2} n^{hs},$$

where $n^{hs} = \frac{1}{2}(\frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \frac{\lambda\theta n_B^{D^e} - \lambda\theta n_A^{D^e}}{2t} + \frac{(1-\lambda)\theta n_B^{D^e} - (1-\lambda)\theta n_A^{D^e}}{2t})$; it is the consumer demand of handset from platform B.

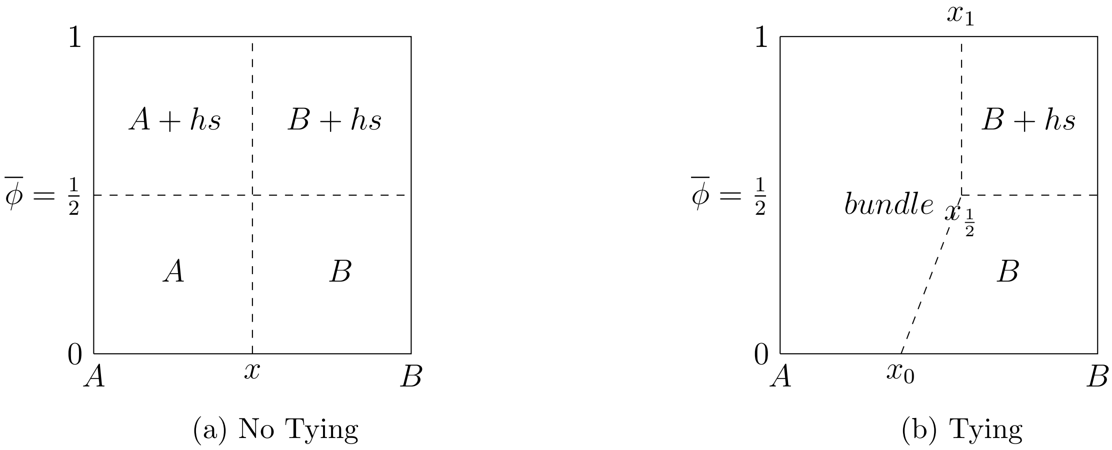


Figure 7: Tying when consumers are heterogeneous w.r.t. the valuation of the handset

Proposition 8. *When platform A practices tying with its in-house handset and consumer's marginal utility of the handset quality is uniformly distributed over* $[0,1]$*, the equilibrium outcomes are as follows:*

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(20t - 4\theta\beta - 3\beta^2 - 4\theta^2\lambda - 9\theta\beta\lambda)}{16(6t - 6\underline{t})},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{16(6t - 6\underline{t})},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}, \qquad n_B^{C^*} = \frac{1}{2} - \frac{z}{8(6t - 6\underline{t})},$$

$$p_A^{D^*} = \frac{\beta - \theta\lambda}{2}\left(\frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}\right), \qquad n_A^{D^*} = \frac{\beta + \theta\lambda}{2}\left(\frac{1}{2} + \frac{z}{8(6t - 6\underline{t})}\right),$$

$$p_B^{D^*} = \frac{\beta - \theta\lambda}{2}\left(\frac{1}{2} - \frac{z}{8(6t - 6\underline{t})}\right), \qquad n_B^{D^*} = \frac{\beta + \theta\lambda}{2}\left(\frac{1}{2} - \frac{z}{8(6t - 6\underline{t})}\right),$$

$$n^{hs^*} = \frac{1}{4} - \frac{z(8t - 6\underline{t})}{32t(6t - 6\underline{t})},$$

$$\pi_A^* = (p_A^{C^*} + \frac{z}{2})n_A^{C^*} + p_A^{D^*} n_A^{D^*} + n^{hs^*}\frac{z}{2}$$

and

$$\pi_B^* = \frac{(8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda)}{256}\frac{(24t - 24\underline{t} - z)^2}{(6t - 6\underline{t})^2}.$$

Proof. See Appendix. □

Corollary 5. *Under tying, the implicit consumer subscription price of platform A, hence the price for the bundle, is higher relative to the bundling case, its consumer demand for the bundle and developer subscription price as well as developer participation are lower relative to the bundling case. Platform A makes less profit through subscription under tying than bundling. Tying also hurts the rival, but platform B is better off than under bundling.*

Under tying, platform A has fewer incentives to offer a discount on consumer subscription price relative to the bundling case. This is because the demand from consumers with high marginal utility of the handset acts less sensitively to a fall of p_A^C. Under bundling, when consumers with a high valuation of the handset choose between the bundle and platform B, a discount on subscription price and the utility from consuming the handset make the bundle more attractive among these consumers. Bundling induces more consumers with a high valuation of the handset to purchase the bundle ($\Delta n^C_{A(\phi \geq \overline{\phi})} = n^C_{A(\phi \geq \overline{\phi})}(bundling) - n^C_{A(\phi \geq \overline{\phi})}(tying) = \frac{z(12t-6\underline{t})}{32t(6t-6\underline{t})} > 0$). This suggests that, relative to tying scheme, bundling is more effective not only to stimulate consumer demand but also to target certain consumers for participation.

6 Concluding Remarks

This work studies how bundling practice and the level of consumer information about developer subscription prices affect platform competition. In this paper, bundling is a commitment to an aggressive pricing strategy; it is deployed to stimulate consumer demand. We show that bundling can be beneficial to the bundling platform and detrimental to the rival when platforms engage in divide-and-conquer strategies given consumers are homogeneous with respect to the valuation of the bundling handset. Once we assume that consumers are heterogeneous with respect to the valuation of the handset, the set of parameters upon which bundling emerges is strictly larger than the previous case. Bundling is more effective to target consumers with a high valuation of the handset. A larger fraction of informed consumers intensifies price competition. Informed consumers respond to price changes by adjusting their own demand as well as the expectation of developer demand. This amplifies the effect of a discount on consumer subscription prices. Therefore, bundling is more effective to stimulate consumer demand when there are more informed consumers. Bundling is less likely to emerge when there is a larger fraction of informed consumers. We further show that bundling and more information increase consumer welfare by lowering subscription prices and improving platform quality in terms of application variety.

Our results offer clear strategy and policy recommendations. From a strategy perspective, both platforms have incentives to affect consumer's knowledge regarding developer subscription prices. Both platforms have incentives to withhold the information because a high level of consumer information intensifies price competition on both sides. Also, bundling is less likely to occur when there is a higher level of consumer information. However, when bundling does occur, the two platforms may have different attitudes towards consumer information. The bundling platform prefers a high level of consumer information because bundling is more effective to stimulate consumer demand. The competing platform wishes to withhold the information as it gets worse off as the level of consumer information increases. Because bundling works as a commitment to an aggressive pricing strategy and it emerges when the platforms subsidize consumers for participation, this work shows that bundling can be used as a tool to enhance the "divide-and-conquer" nature of pricing strategies.

From a public policy perspective, our results concern bundling and information disclosure. In conventional one-sided markets, bundling is usually considered to be anti-competitive by competition authorities as it's adopted either for price discrimination or foreclosure reasons, but analyzing a two-sided market using one-sided market logic may lead to policy errors (Wright, 2004). Due to the existence of (positive) network externalities, consumer surplus increases with the number of developers on the same platform. Bundling does not affect only the consumer subscription prices but also the perceived quality of platforms as it affects developer participation. We have shown that pure bundling improves consumer welfare mainly because it offers a lower subscription price and more application variety to the majority of consumers. For the same reason, even when bundling implements second-degree price discrimination, bundling still improves consumer welfare. Also, information disclosure unambiguously improves consumer surplus by lowering subscription prices on both sides of the platform and improving developer participation. Thus, information disclosure should be encouraged or mandated for consumer's sake.

Appendix A Appendix

A.1 Proof of Proposition 1

A fraction λ of consumers is informed about developer subscription prices and holds responsive expectations, while the remaining fraction $1-\lambda$ of consumers is uninformed about developer prices and holds passive expectations, the consumer demand for platform T is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t} + \lambda\frac{\theta n_T^D - \theta n_{-T}^D}{2t} + (1-\lambda)\frac{\theta n_T^{D^e} - \theta n_{-T}^{D^e}}{2t}, \tag{8}$$

and the developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D, \tag{9}$$

where $T = A, B$. As the fraction λ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.9) to Eq. (1.8) for n_T^D and n_{-T}^D, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations on developer demand of each platform. The realized consumer demand of platform T is

$$n_T^C = \frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda},$$

and the developer demand is

$$n_T^D = \beta(\frac{1}{2} + \frac{p_{-T}^C - p_T^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_T^{D^e} - \theta(1-\lambda)n_{-T}^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_{-T}^D - \theta\lambda p_T^D}{2t - 2\theta\beta\lambda}) - p_T^D.$$

Platform T's profit maximization problem is

$$\max_{p_T^C, p_T^D} \quad \pi_T = p_T^C n_T^C + p_T^D n_T^D.$$

Taking the first order conditions of the profit function in p_T^C and p_T^D and solving for p_T^C and p_T^D as functions of uninformed consumers' expectations on developer demands, we obtain:

$$p_T^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2} \tag{10}$$

and

$$p_T^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}. \tag{11}$$

Substituting Eqs. (1.10) and (1.11) to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations on developer demand:

$$n_T^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2}$$

and

$$n_T^D = \frac{(\beta+\theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_T^{D^e} - n_{-T}^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_T^D = n_T^{D^e}$, we obtain equilibrium prices and allocations:

$$p_T^{C*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4}, \qquad n_T^{C*} = \frac{1}{2},$$

$$p_T^{D*} = \frac{\beta}{4} - \frac{\theta\lambda}{4}, \qquad \text{and} \qquad n_T^{D*} = \frac{\beta}{4} + \frac{\theta\lambda}{4}.$$

A.2 Proof of Proposition 2

Platform A sets the price for the bundle $p_A = p_A^C + p^{hs}$, where $p^{hs} = z$. The marginal consumer locates at

$$x = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

Therefore, the consumer demands are

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}, \tag{12}$$

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \lambda\frac{\theta n_B^D - \theta n_A^D}{2t} + (1-\lambda)\frac{\theta n_B^{D^e} - \theta n_A^{D^e}}{2t}. \tag{13}$$

The developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D. \tag{14}$$

As the fraction λ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.14) to Eqs. (1.12) and (1.13) for n_A^D and n_B^D, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations on developer demands:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_A^{D^e} - \theta(1-\lambda)n_B^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_B^D - \theta\lambda p_A^D}{2t - 2\theta\beta\lambda},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{2t - 2\theta\beta\lambda}.$$

Platform A's profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D = (p_A^C + z)n_A^C + p_A^D n_A^D.$$

Platform B's profit maximization problem remains the same.
Taking the first order conditions of the profit function in p_T^C and p_T^D and solving for p_T^C and p_T^D as functions of uninformed consumers' expectations, we obtain:

$$\begin{aligned} p_A^C = & \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2} \\ & - \frac{(8t - \beta^2 - 2\theta^2\lambda^2 - 5\theta\beta\lambda)z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2}, \end{aligned} \tag{15}$$

$$p_B^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{16}$$

$$p_A^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}, \tag{17}$$

and

$$p_T^D = \frac{(\beta - \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}. \tag{18}$$

Substituting Eqs. (1.15) to (1.18) four functions of subscription prices to demand functions, we obtain demand functions on both sides of the market as functions of uninformed consumers' expectations on developer demand:

$$n_A^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_B^C = \frac{6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z}{12t - 2\beta^2 - 8\theta\beta\lambda - 2\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 4\theta\beta\lambda + 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta + \theta\lambda)(6t - \beta^2 - \theta^2\lambda^2 + 2\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 4\theta\beta\lambda - 2z)}{24t - 4\beta^2 - 16\theta\beta\lambda - 4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z}{2}\frac{(8t - 2\theta\beta - \beta^2 - 2\theta^2\lambda - 3\theta\beta\lambda)}{(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z}{2}\frac{(4t - \beta^2 - 3\theta\beta\lambda)}{(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda}),$$

$$n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda}),$$

$$n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda}).$$

A.3 Proof of Proposition 3

Let $z_1 = \frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)(16t+\theta^2\lambda^2-4\theta^2\lambda-6\theta\beta\lambda-4\theta\beta-3\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$, $t_1 = \frac{3\beta^2+4\theta\beta+6\theta\beta\lambda+4\theta^2\lambda-\theta^2\lambda^2}{16}$, and $t_2 = \frac{5\beta^2+8\theta\beta+6\theta\beta\lambda+8\theta^2\lambda-3\theta^2\lambda^2}{24}$.

When consumers are homogeneous with respect to the valuation of the handset, bundling leads to change on platform A's profit

$$\begin{aligned}\Delta\pi_A &= \pi_A^{BUN} - \pi_A^{NB} \\ &= -\frac{z}{4}(\frac{16t + \theta^2\lambda^2 - 4\theta^2\lambda - 6\theta\beta\lambda - 4\theta\beta - 3\beta^2}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda} - \frac{(8t - \theta^2\lambda^2 - 6\theta\beta\lambda - \beta^2)z}{(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)^2}).\end{aligned}$$

Platform A would choose bundling iff $\Delta\pi_A \geq 0 \Rightarrow z \geq z_1$.
Either (i) $z_1 < 0$ or
(ii) $0 < z_1 < \overline{z}$ holds.
(i) When $\underline{t} < t \leq t_1$, $z_1 \leq 0$, $z > z_1$ holds as $z > 0$.
Therefore, when $\underline{t} < t \leq t_1$, $\Delta\pi_A > 0$.
(ii) When $t_1 < t \leq t_2$, $0 < z_1 < \overline{z}$ holds. $\Delta\pi_A \geq 0$ when $z_1 \leq z < \overline{z}$.
When $t > t_2$, $\Delta\pi_A \geq 0$ when $z \geq z_1$, which contradicts assumption $A3$.

Bundling leads to change on platform B's profit $\Delta\pi_B = -\frac{z(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2)}{4}\frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2}$.
As $\Delta p_B^D = -\frac{z(\beta-\theta\lambda)}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)} < 0$ and $\Delta n_B^D = -\frac{z(\beta+\theta\lambda)}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)} < 0$.
$\Delta\pi_B^D = -\frac{z(\beta^2-\theta^2\lambda^2)}{4}\frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2} < 0$. Therefore, $(6t - \beta^2 - \theta\beta - \theta^2\lambda - 3\theta\beta\lambda - z) > 0$.
As $\pi_B^* = \frac{8t-\beta^2-\theta^2\lambda^2-6\theta\beta\lambda}{16}\frac{(6t-\theta\beta-\beta^2-\theta^2\lambda+3\theta\beta\lambda-2z)^2}{(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)^2}$, $8t - \beta^2 - \theta^2\lambda^2 - 6\theta\beta\lambda > 0$ also holds.
Therefore, $\Delta\pi_B < 0$.

A.4 Proof of Proposition 6

Platform A sets the price for the bundle $p_A = p_A^C + p^{hs}$, where $p^{hs} = \frac{z}{2}$. The marginal consumer of type $(x, 0)$, whose marginal utility of the handset quality $\phi = 0$, locates at

$$x_0 = \frac{1}{2} - \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

The marginal consumer with the highest marginal utility of the handset quality $\phi = 1$, locates at

$$x_1 = \frac{1}{2} + \frac{z}{4t} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}.$$

The consumer demands for platform A is $n_A^C = \frac{x_0 + x_1}{2}$. Therefore, the realized consumer demands are:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1-\lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}, \quad (19)$$

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t} + \lambda\frac{\theta n_B^D - \theta n_A^D}{2t} + (1-\lambda)\frac{\theta n_B^{D^e} - \theta n_A^{D^e}}{2t}. \quad (20)$$

The developer demand of each platform is

$$n_T^D = \beta n_T^C - p_T^D, \quad (21)$$

where $T = A, B$. As the fraction λ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.21) to Eqs. (1.19) and (1.20) for n_A^D and n_B^D, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations:

$$n_A^C = \frac{1}{2} + \frac{p_B^C - p_A^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_A^{D^e} - \theta(1-\lambda)n_B^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_B^D - \theta\lambda p_A^D}{2t - 2\theta\beta\lambda},$$

and

$$n_B^C = \frac{1}{2} + \frac{p_A^C - p_B^C}{2t - 2\theta\beta\lambda} + \frac{\theta(1-\lambda)n_B^{D^e} - \theta(1-\lambda)n_A^{D^e}}{2t - 2\theta\beta\lambda} + \frac{\theta\lambda p_A^D - \theta\lambda p_B^D}{2t - 2\theta\beta\lambda}.$$

Platform A's profit maximization problem is

$$\max_{p_A^C, p_A^D} \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D = (p_A^C + \frac{z}{2})n_A^C + p_A^D n_A^D.$$

Platform B's profit maximization problem remains the same.
Taking the first order conditions of the profit function in p_T^C and p_T^D and solving for p_T^C and

p_T^D as functions of uninformed consumers' expectations on developer demands, we obtain:

$$\begin{aligned} p_A^C = & \frac{(4t-\beta^2-3\theta\beta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_A^{D^e}-n_B^{D^e})-4\theta\beta\lambda)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2} \\ & -\frac{(8t-\beta^2-2\theta^2\lambda^2-5\theta\beta\lambda)z}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}, \end{aligned} \tag{22}$$

$$p_B^C = \frac{(4t-\beta^2-3\theta\beta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_B^{D^e}-n_A^{D^e})-4\theta\beta\lambda-z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}, \tag{23}$$

$$p_A^D = \frac{(\beta-\theta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_A^{D^e}-n_B^{D^e})-4\theta\beta\lambda+z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}, \tag{24}$$

and

$$p_B^D = \frac{(\beta-\theta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_B^{D^e}-n_A^{D^e})-4\theta\beta\lambda-z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}. \tag{25}$$

Substituting Eqs. (1.22) to (1.25) four price functions to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations:

$$n_A^C = \frac{6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_A^{D^e}-n_B^{D^e})-4\theta\beta\lambda+z}{12t-2\beta^2-8\theta\beta\lambda-2\theta^2\lambda^2},$$

$$n_B^C = \frac{6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_B^{D^e}-n_A^{D^e})-4\theta\beta\lambda-z}{12t-2\beta^2-8\theta\beta\lambda-2\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta+\theta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_A^{D^e}-n_B^{D^e})-4\theta\beta\lambda+z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta+\theta\lambda)(6t-\beta^2-\theta^2\lambda^2+2\theta(1-\lambda)(n_B^{D^e}-n_A^{D^e})-4\theta\beta\lambda-z)}{24t-4\beta^2-16\theta\beta\lambda-4\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(8t-2\theta\beta-\beta^2-2\theta^2\lambda-3\theta\beta\lambda)}{4(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t-\beta^2-3\theta\beta\lambda)}{4(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{2(6t-\theta\beta-\beta^2-\theta^2\lambda-3\theta\beta\lambda)},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{2(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}).$$

A.5 Proof of Proposition 7

Let $z_2 = \frac{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)(8t+2\theta^2\lambda^2-4\theta^2\lambda-4\theta\beta-2\beta^2)}{8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2}$, $t_3 = \frac{\beta^2+2\theta\beta+2\theta^2\lambda-\theta^2\lambda^2}{4}$, and $t_4 = \frac{3\beta^2+8\theta\beta-6\theta\beta\lambda+8\theta^2\lambda-5\theta^2\lambda^2}{8}$.
When consumer's valuation of the handset is uniformly distributed along $[0, 1]$, bundling leads to change on platform A's profit

$$\begin{aligned}\Delta\pi_A &= \pi_A^{BUN} - \pi_A^{NB} \\ &= -\frac{z}{16}(\frac{8t + 2\theta^2\lambda^2 - 4\theta^2\lambda - 4\theta\beta - 2\beta^2}{6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda} - \frac{8t - \theta^2\lambda^2 - 6\theta\beta\lambda - \beta^2)z}{(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)^2}).\end{aligned}$$

Platform A would choose bundling iff $\Delta\pi_A \geq 0 \Rightarrow z \geq z_2$.
Either (i) $z_2 < 0$ or
(ii) $0 < z_2 < \overline{z}$ holds.
(i) When $\underline{t} < t \leq t_3$, $z_2 < 0$, $z > z_2$ holds as $z > 0$. Therefore, when $\underline{t} < t \leq t_3$, $\Delta\pi_A > 0$.
(ii) When $t_3 < t \leq t_4$, $0 < z_2 < \overline{z}$ holds. $\Delta\pi_A \geq 0$ when $z_2 \leq z < \overline{z}$.

Bundling leads to change on platform B's profit
$\Delta\pi_B = -\frac{z(8t-\theta^2\lambda^2-6\theta\beta\lambda-\beta^2)}{16}\frac{(12t-2\beta^2-2\theta\beta-2\theta^2\lambda-6\theta\beta\lambda-z)}{(6t-\beta^2-\theta\beta-\theta^2\lambda-3\theta\beta\lambda)^2}$.
$(8t - \theta^2\lambda^2 - 6\theta\beta\lambda - \beta^2) > 0$ and $(12t - 2\beta^2 - 2\theta\beta - 2\theta^2\lambda - 6\theta\beta\lambda - z) > 0$,therefore, $\Delta\pi_B < 0$.

A.6 Proof of Proposition 8

Proof of Proposition 8
Consumers with high marginal utility of the handset quality, i.e., $\phi \geq \overline{\phi} = \frac{1}{2}$, have the demand for the bundle from platform A

$$n_{A(\phi \geq \frac{1}{2})}^C = \frac{1}{2}(\frac{1}{2} + \frac{p_B^C - p_A^C}{2t} + \lambda\frac{\theta n_A^D - \theta n_B^D}{2t} + (1 - \lambda)\frac{\theta n_A^{D^e} - \theta n_B^{D^e}}{2t}),$$

and the demand for access to platform B plus handset from platform A

$$n^C_{B(\phi\geq\frac{1}{2})}=\frac{1}{2}(\frac{1}{2}+\frac{p^C_A-p^C_B}{2t}+\lambda\frac{\theta n^D_B-\theta n^D_A}{2t}+(1-\lambda)\frac{\theta n^{D^e}_B-\theta n^{D^e}_A}{2t}).$$

Consumers with low marginal utility of the handset quality, i.e., $\phi<\overline{\phi}=\frac{1}{2}$, have the demand for the bundle from platform A

$$n^C_{A(\phi<\frac{1}{2})}=\frac{1}{2}(\frac{1}{2}+\frac{p^C_B-p^C_A}{2t}-\frac{z}{8t}+\lambda\frac{\theta n^D_A-\theta n^D_B}{2t}+(1-\lambda)\frac{\theta n^{D^e}_A-\theta n^{D^e}_B}{2t}),$$

and the demand for access to platform B plus handset from platform A

$$n^C_{B(\phi<\frac{1}{2})}=\frac{1}{2}(\frac{1}{2}+\frac{p^C_A-p^C_B}{2t}+\frac{z}{8t}+\lambda\frac{\theta n^D_B-\theta n^D_A}{2t}+(1-\lambda)\frac{\theta n^{D^e}_B-\theta n^{D^e}_A}{2t}).$$

There, the aggregate consume demand for the bundle is

$$n^C_A=\frac{1}{2}+\frac{p^C_B-p^C_A}{2t}-\frac{z}{16t}+\lambda\frac{\theta n^D_A-\theta n^D_B}{2t}+(1-\lambda)\frac{\theta n^{D^e}_A-\theta n^{D^e}_B}{2t},\tag{26}$$

and for platform B is

$$n^C_B=\frac{1}{2}+\frac{p^C_A-p^C_B}{2t}+\frac{z}{16t}+\lambda\frac{\theta n^D_B-\theta n^D_A}{2t}+(1-\lambda)\frac{\theta n^{D^e}_B-\theta n^{D^e}_A}{2t}.\tag{27}$$

The developer demand of each platform is

$$n^D_T=\beta n^C_T-p^D_T,\tag{28}$$

where $T=A,B$. As the fraction λ of consumers is informed about the developer prices and the structure of developer demand, we substitute Eq. (1.28) to Eqs. (1.26) and (1.27) for n^D_A and n^D_B, and solve for consumer demand as a function of subscription prices on both sides of the platform and uninformed consumers' expectations:

$$n^C_A=\frac{1}{2}+\frac{p^C_B-p^C_A}{2t-2\theta\beta\lambda}-\frac{z}{16t-16\theta\beta\lambda}+\frac{\theta(1-\lambda)n^{D^e}_A-\theta(1-\lambda)n^{D^e}_B}{2t-2\theta\beta\lambda}+\frac{\theta\lambda p^D_B-\theta\lambda p^D_A}{2t-2\theta\beta\lambda},$$

and

$$n^C_B=\frac{1}{2}+\frac{p^C_A-p^C_B}{2t-2\theta\beta\lambda}+\frac{z}{16t-16\theta\beta\lambda}+\frac{\theta(1-\lambda)n^{D^e}_B-\theta(1-\lambda)n^{D^e}_A}{2t-2\theta\beta\lambda}+\frac{\theta\lambda p^D_A-\theta\lambda p^D_B}{2t-2\theta\beta\lambda}.$$

Also, the demand for the handset from consumers on platform B is

$$n^{hs}_B=\frac{1}{4}+\frac{p^C_A-p^C_B}{4t-4\theta\beta\lambda}+\frac{z}{32t^2-32t\theta\beta\lambda}+\frac{\theta(1-\lambda)n^{D^e}_B-\theta(1-\lambda)n^{D^e}_A}{4t-4\theta\beta\lambda}+\frac{\theta\lambda p^D_A-\theta\lambda p^D_B}{4t-4\theta\beta\lambda}.$$

Platform A's profit maximization problem is

$$\max_{p_A^C, p_A^D} \quad \pi_A = (p_A^C + p^{hs})n_A^C + p_A^D n_A^D + p^{hs} n_B^{hs} = (p_A^C + \frac{z}{2})n_A^C + p_A^D n_A^D + \frac{z}{2} n_B^{hs}.$$

Platform B's profit maximization problem remains the same.
Taking the first order conditions of the profit function in p_T^C and p_T^D and solving for p_T^C and p_T^D as functions of uninformed consumers' expectations on developer demands, we obtain:

$$p_A^C = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} + \frac{(4t - \beta^2 - 3\theta\beta\lambda)(8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e})}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2} + \frac{(20t - 3\beta^2 - 4\theta^2\lambda^2 - 13\theta\beta\lambda)z}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{29}$$

$$p_B^C = \frac{(4t - \beta^2 - 3\theta\beta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 - 16\theta\beta\lambda + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{30}$$

$$p_A^D = \frac{(\beta - \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}, \tag{31}$$

and

$$p_B^D = \frac{(\beta - \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 16\theta\beta\lambda - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}. \tag{32}$$

Substituting Eqs. (1.29) to (1.32) four price functions to demand functions, we obtain demand functions on both sides of the platform as functions of uninformed consumers' expectations:

$$n_A^C = \frac{24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z}{48t - 8\beta^2 - 32\theta\beta\lambda - 8\theta^2\lambda^2},$$

$$n_B^C = \frac{24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_B^{D^e} - n_A^{D^e}) - 16\theta\beta\lambda - z}{48t - 8\beta^2 - 32\theta\beta\lambda - 8\theta^2\lambda^2},$$

$$n_A^D = \frac{(\beta + \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda + z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2},$$

and

$$n_B^D = \frac{(\beta + \theta\lambda)(24t - 4\beta^2 - 4\theta^2\lambda^2 + 8\theta(1-\lambda)(n_A^{D^e} - n_B^{D^e}) - 16\theta\beta\lambda - z)}{96t - 16\beta^2 - 64\theta\beta\lambda - 16\theta^2\lambda^2}.$$

In equilibrium, uninformed consumers' expectations are fulfilled. Imposing $n_A^D = n_A^{D^e}$ and $n_B^D = n_B^{D^e}$, we obtain equilibrium prices and allocations:

$$p_A^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(20t - 4\theta\beta - 3\beta^2 - 4\theta^2\lambda - 9\theta\beta\lambda)}{16(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_B^{C^*} = t - \frac{\beta^2}{4} - \frac{3\theta\beta\lambda}{4} - \frac{z(4t - \beta^2 - 3\theta\beta\lambda)}{16(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_A^{C^*} = \frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$n_B^{C^*} = \frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)},$$

$$p_A^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_A^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} + \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$p_B^{D^*} = (\frac{\beta - \theta\lambda}{2})(\frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)}),$$

$$n_B^{D^*} = (\frac{\beta + \theta\lambda}{2})(\frac{1}{2} - \frac{z}{8(6t - \theta\beta - \beta^2 - \theta^2\lambda - 3\theta\beta\lambda)})$$

and

$$n^{hs^*} = \frac{1}{4} - \frac{z(8t - 6\underline{t})}{32t(6t - 6\underline{t})}.$$

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